Estimation of anthropometrical and inertial body parameters

An example for squat jump

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Content

Abstract

The purpose of this study was to optimize inertial (IP) and anthropometrical (AP) parameters of human body in order to minimize the residual torque and force during squat jumping.
Three methods of determination have been presented: method A: optimizing AP and IP of each body part, method B: optimizing trunk AP and IP, assuming that the AP and IP of the lower limbs were known, method C: using Winter AP and IP. For each method, the value (degree 0), the integral (degree 1) and the double integral (degree 2) of the residual moment were also used. The method B with degree 2 was the most accurate to determine trunk AP and IP by minimizing of the residual force and torque, by providing a linear least square system. Instead of minimizing the residual force and torque, by classical way, the double integral of the latter provided more accurate results.
These results come out from [BBM13].

Content

Objectives and Assumptions

- Planar polyarticulated system;
- Rigid segments;
- A simple movement is studied (squat jump).

We suppose that the length of segments are known and we try to determine all or a part of the inertial (IP) and anthropometrical (AP) parameters, by using the measured displacements and the measured actions to the subject of the ground (force and torque). The residual forces and torques are minimized to determine IP and AP parameters. Instead of minimizing the residual force and torque, by classical way, the double integral of the latter provided more accurate results.

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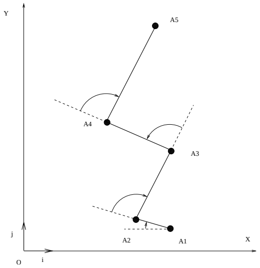
Definition of subjects segments

During the squat jump, the trunk, the arms and head are assumed to move together. $\Longrightarrow$ the considered segments are:

- the "head-harm-trunk" segment (HAT);
- the shank;
- the thigh;
- and the foot.

The IP and AP and defined by $\alpha_j = A_jG_j/A_jA_{j+1}$ and the normalized radius of giration $\widetilde{r}_j = r_j/A_jA_{j+1}$. α_j, for $1 \leq j \leq 3$, are given by using Winter's data.



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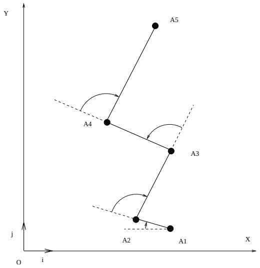

Content

Preparation

- Coordinates of points A_j are determined with a camcorder and the force and torque action of the ground $\vec{R}$ and C are determined with a force plate.
- The data are synchronized and α_4 are deduced from them (by minimizing the residual force).
- The displacement data are smoothed. Then the velocity and the acceleration of the joint articulations can be calculated.

Determination of I_j

We have:

$$\vec{R}_j - \vec{R}_{j+1} - = -m_j \vec{g} + m_j \frac{d^2 \overrightarrow{OG_j}}{dt^2}, \tag{1a}$$

$$-\mathcal{M}_j + I_j \ddot{\phi}_j = C_j - C_{j+1}, \tag{1b}$$

where

$$\mathcal{M}_j = -(x_{j+1} - x_j)\left(\alpha_j R_{y,j} + (1-\alpha_j) R_{y,j+1}\right) + (y_{j+1} - y_j)\left(\alpha_j R_{x,j} + (1-\alpha_j) R_{x,j+1}\right), \tag{2}$$

with boundary conditions

$$\vec{R}_1 = \vec{R}, \quad \vec{R}_p = \vec{0}, \quad C_1 = C, \quad C_q = 0. \tag{3}$$

Determination of I_j

We obtain classically (see [Hof92]), for all $k \in \{1, \ldots, q-1\}$,

$$\vec{R}_k = \vec{R} - \sum_{j=1}^{k-1} m_j \left(\frac{d^2 \overrightarrow{OG_j}}{dt^2} - \vec{g} \right), \tag{4a}$$

$$C_k = C + \sum_{j=1}^{k-1} \left(\mathcal{M}_j - I_j \ddot{\phi}_j \right), \tag{4b}$$

$$C = -\sum_{j=1}^{q-1} \mathcal{M}_j + \sum_{j=1}^{q-1} I_j \ddot{\phi}_j. \tag{4c}$$

Determination of I_j

The residual torque is defined by

$$\widetilde{C} = C + \sum_{j=1}^{q-1} \mathcal{M}_j - \sum_{j=1}^{q-1} I_j \ddot{\phi}_j, \tag{5}$$

where angles ϕ_j are determined from the smoothed displacements, $\mathcal{M}_j$ are defined by (2) and joint forces $R_{x,j}$ et $R_{y,j}$ are calculated by using (4a).

Determination of I_j (order 0)

The residual torque defined by (5) is in theory equal to zero, but it is in fact non experimentally equal to zero. We try then to choose I_j to minimize this residual torque [RHW08; RHW09; Kuo98; Vau+82]. We write then

$$\widetilde{C}^{(0)}(t) = C_{\text{exp}} - C_{\text{angl}}, \tag{6a}$$

where C_{exp} is torque measured experimentally and

$$C_{\text{angl}} = -\sum_{j=1}^{q-1} \mathcal{M}_j + \sum_{j=1}^{q-1} I_j \ddot{\phi}_j, \tag{6b}$$

is defined according to moments $\mathcal{M}_j$ and the double derivatives $\ddot{\phi}_j$. $X^{(0)}$ corresponds to the values of function X. The impulsion phase is equal to $[t_0, t_f]$.

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Determination of I_j (order 1)

By integration, between the beginning t_0 and t_i, we obtain,

$$\widetilde{C}^{(1)}(t_i) = C^{(1)}_{\text{exp}}(t_i) - C^{(1)}_{\text{angl}}(t_i), \tag{7a}$$

$$C^{(1)}_{\text{exp}}(t_i) = \int_{t_0}^{t_i} C_{\text{exp}}(s)ds, \tag{7b}$$

$$C^{(1)}_{\text{angl}}(t_i) = -\sum_{j=1}^{q-1} \int_{t_0}^{t_i} \mathcal{M}_j(s)ds + \sum_{j=1}^{q-1} I_j \dot{\phi}_j(t_i). \tag{7c}$$

$X^{(1)}$ corresponds to the first order integration of the function X.

Determination of I_j (order 2)

After a second integration we obtain:

$$\widetilde{C}^{(2)}(t_i) = C^{(2)}_{\exp}(t_i) - C^{(2)}_{\text{angl}}(t_i), \tag{8a}$$

$$C^{(2)}_{\exp}(t_i) = \int_{t_0}^{t_i} \int_{t_0}^{u} C_{\exp}(s) ds du, \tag{8b}$$

$$C^{(2)}_{\text{angl}}(t_i) = -\sum_{j=1}^{q-1} \int_{t_0}^{t_i} \int_{t_0}^{u} \mathcal{M}_j(s) ds du + \sum_{j=1}^{q-1} I_j \left(\phi_j(t_i) - \phi_j(t_0)\right). \tag{8c}$$

$X^{(2)}$ corresponds to the second order integration of the function X.

Determination of l_j

We obtain then 3 orders of methods, defined by (6), (7) and (8).
In order to compare the residual values $\widetilde{C}^{(0)}$, $\widetilde{C}^{(1)}$ and $\widetilde{C}^{(2)}$ obtained with different methods, it is necessary to normalize these values by considering the dimensionless quantity defined by

$$\varepsilon^{(j)} = \frac{\left\| C_{\mathrm{exp}}^{(j)} - C_{\mathrm{ang}}^{(j)} \right\|}{\left\| C_{\mathrm{exp}}^{(j)} \right\| + \left\| C_{\mathrm{ang}}^{(j)} \right\|} \in [0,1], \tag{9}$$

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Determination of I_j, $1 \leq j \leq 4$ (method A)

If we consider that I_j, $1 \leq j \leq 4$ are unknow, the equations (6), (7) and (8) are equivalent to determine I_1, I_2, I_3 and I_4 such that

$$\forall i, \quad \sum_{j=1}^{q-1} A_{i,j} I_j = B_i \tag{10}$$

where $A_{i,j}$ and B_i are known. These equations are equivalent to the overdetermined linear system

$$AI = B, \text{ where } I = \begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{pmatrix} \tag{11}$$

which has no solution in the general case, but has a least square sense solution [LT93]. For each order $j \in \{0, 1, 2\}$, we have method "Aj".

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Determination of l_4 alone (method B)

If we consider that l_j, $1 \leq j \leq 3$ are know (from Winter) and l_4 only unknow, the equations (6), (7) and (8) are equivalent to determine l_4 such that

$$\forall i, \quad y_i = l_4 x_i. \tag{12}$$

where x_i and y_i are known. These equations are equivalent to the overdetermined linear system (11). For each order $j \in \{0, 1, 2\}$, we have method "Bj".

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Fixed values of I_j, $1 \leq j \leq 4$ (method C)

If we consider that I_j, $1 \leq j \leq 4$ are know (from Winter). Then, the equations (6), (7) and (8) are not a least square linear system. But, this method, for each order $j \in \{0, 1, 2\}$, is called method "Cj".

Comparison between the 9 methods

- To summarize, we have three methods defined by $X \in \{A, B, C\}$ and for each of them the order j belongs to $j \in \{0, 1, 2\}$. The method X with degree j is called method "Xj".
- For each of these three methods and for each degree j are defined $\varepsilon_X^{(j)}$ and $R2_X^{(j)}$, which is the coefficient of multiple determination for the overdetermined system (11)
- An accurate method corresponds to ε, close to 0 and R^2 close to 1.

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Content

Generalization on the population (twelve subjets)

12 subjects performed beetwen 5 and 10 (mean: 7.25) maximal squat jumps, which provided 97 squat jumps. The non positive or greater than 1 values of radius of gyration were removed. Then, the number kept for analysis was 87.

The Shapiro-Wilk test shows that data $\varepsilon_X^{(j)}$ and $1 - R^2{}_X^{(j)}$ do not present a normal distribution; on the contrary, the logarithm of these data follow a Gaussian distribution.

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Groups statistics

Groups statistics of $\log_{10}(\varepsilon_{\mathrm{X}}^{(j)})$:

degree j	method A	method B	method C
0	-0.46 ± 0.16	-0.35 ± 0.14	-0.28 ± 0.1
1	-1.06 ± 0.29	-0.59 ± 0.3	-0.37 ± 0.25
2	-1.83 ± 0.44	-1.01 ± 0.38	-0.49 ± 0.33

Groups statistics of $\log_{10}\left(1 - R^{2}{}_{\mathrm{X}}^{(j)}\right)$:

degree j	method A	method B	method C
0	-0.27 ± 0.28	-0.05 ± 0.23	0.11 ± 0.24
1	-1.47 ± 0.48	-0.52 ± 0.31	0.03 ± 0.54
2	-2.99 ± 0.73	-1.37 ± 0.56	-0.2 ± 0.82

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Conclusion

We obtain

$$A2 < B2 = A1 < B1 < C2 < C0, \tag{13a}$$
$$B2 < B1 < B0, \tag{13b}$$
$$A0 < B0, \tag{13c}$$

that confirms the results for one subject (14). As shown previously, the results of method A gave unphysical values. Therefore the most accurate method was the method B2.

Inverse dynamics

Remark

Since I_j are known, Eq. (4b) allows us to determine torques C_k (results of inverse dynamics).

We now give results for one subject.

Method A

case	degree $j = 0$	degree $j = 1$	degree $j = 2$
I_1	-21.108	9.964	13.133
I_2	-9.679	10.139	12.557
I_3	-1.563	0.585	1.296
I_4	-3.633	8.289	9.527

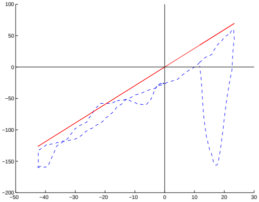
The calculated IP are presented above. This method is not valuable while it provides values without any physical meaning. Indeed, we can see that:

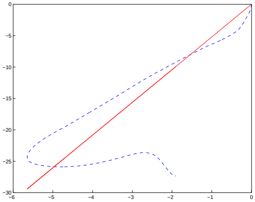
- For $j = 0$, obtained values are not positive.
- For $j = 1$ or $j = 2$, obtained values differ greatly from values given by Winter.

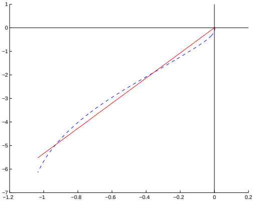
Then, this method has to be removed.

Method B

I_j are determined with Winter's data.
Points (x_i, y_i) for different degrees; The three following figures correspond to $j = 0$, $j = 1$ and $j = 2$.







Method C

For this method, inertia are chosen according to Winter (method C).

Comparison between the three methods and the three degrees and conclusion

Values of $\varepsilon_X^{(j)}$ for different cases:

degree j	method A	method B	method C
0	0.311	0.386	0.457
1	0.065	0.252	0.259
2	0.003	0.047	0.123

Values of $R^{2}{}_X^{(j)}$ for different cases:

degree j	method A	method B	method C
0	0.549	0.246	0.107
1	0.980	0.6904	0.567
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Comparison bettwen the three methods and the three degrees and conclusion

We deduce then

$$A2 < B2 < A1 < B1, \tag{14a}$$

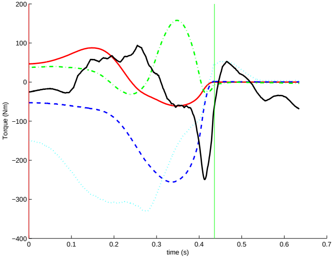
$$B2 < B1 < B0, \tag{14b}$$

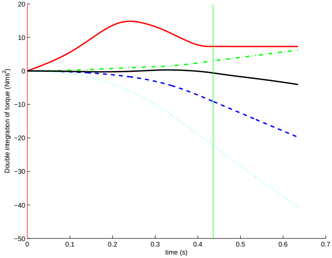
$$C2 < C0, \tag{14c}$$

where "$<$" means "more accurate than".

Inverse dynamic by method B2

Different joint torques and double integration of joint torques.
Residual action is plotted by a black continuous line.





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